Review of "Gaṇitasārakaumudī: The Moonlight of the Essence of Mathematics, by Ṭhakkura Pherū, Edited with Introduction, Translation, and Mathematical Commentary by SaKHYa"

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On setting one's eyes on a copy of the book [1], one is at once struck by "SaKHYa", where one would look for an author-name. Is it indeed an authorname, from an unknown land? Or an institution that takes on the role of author/editor etc.? It turns out, as explained in the Preface, to be a group-name composed from the initial letters of the names of the members of the group, S.R. Sarma, Takanori Kusuba, Takao Hayashi and Michio Yano. It is rather intriguing that together the components add to "sakhya" (friendship); the Preface records a warm bond of friendship that developed among the members of the group through their common interest in history of science. The three Japanese members of the group have worked together on Sanskrit texts for quite a long time. S.R. Sarma joined them in 2002 and the present volume is the fruit of their joint endeavour, completed in 2008, and published a year later.

"Gaṇitasārakaumudī"(hereafter referred to as GSK), composed by Ṭhakkura Pherū around the turn of the thirteenth century, is unique in many respects. It is the first full-edged mathematical text of which the language of composition is Middle Indic (Apabhraṃśa). It deals with a diverse range of topics touching the daily life, well beyond the traditional Sanskrit works. And above all, it reflects on the multicultural ethos of its time, with the Sultanates in place in Delhi, and the Jain community flourishing in trade. The author Ṭhakkura Pherū came from a family of merchant bankers belonging to the Kharatara sect of the Śvetāmbara Jains, hailing from a place identified in Jain literature as Kanyānyana which is now Kaliyana in the Bhiwani district of the present Haryana state. While not much is known of his early life, the evidence from his work suggests that Pherū was born

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around 1270. He was appointed in the treasury of the Khaljī Sulṭāns of Delhi, where he rose to a high position as Treasurer, and served three, and possibly even four, generations of the rulers; "Ṭhakkura" is apparently a title signifying the association with the court of the Sulṭāns. He is seen to have been quite a versatile scholar, having composed authoritative accounts on such varied subjects as gemmology, astronomy, architecture, metallurgy of coins, and metal technology, besides the mathematical work GSK.

Our bequest of the work of Pherū is unfortunately limited to a single manuscript discovered in 1946 by Agar Chand Nahata and Bhanwar Lal Nahata, which they published in 1961, containing seven separate works including a eulogy to the pontiffs of the Kharatara sect. While five of the works have been dated based on internal evidence, variously between 1291 and 1318, for two books, one on the technology of metals and GSK the dates of composition remain uncertain. Taking into account the references to the coinage etc. it could be concluded however that GSK would have been written before 1318.

The book under review is principally an edition of GSK. The authors begin with a detailed introduction, bringing out in a lucid manner the historical background, various special features of Pherū's work in relation to his predecessors both in the Jain and other Indic traditions, significance of the work in understanding the broader aspects of the life and times during the period in question, etc.. Part II documents the original text (transcribed in Roman script) and this is followed in Part III with an English translation; it is interesting to see the neat arrangement of the material, with subheadings indicating the topic under discussion. In Part IV the authors give a detailed, verse by verse, commentary on the mathematical contents of the work; the exposition of the individual mathematical results and principles given in GSK is richly supplemented with comparisons with the corresponding material in other traditional texts. Also, the authors are meticulous in specifying (a few) parts that are not understood, and those where the statement is incorrect, avoiding any fudging or specious explanations. On the whole the commentary provides a scholarly account of the mathematics involved. The reviewer only wishes however that the authors had kept the usage of modern algebraic notation to the minimum; while the usage would indeed make it simpler for a present day reader to understand the contents, it seems to take away some of the flavour of the original, the latter not being algebraical in character.

The four parts mentioned above are followed by various appendices including one, notably, giving a concordance of GSK with many earlier works, namely

Āryabhaṭīya, Brāhmasphuṭasiddhānta, the Bakhshālī manuscript, Pāṭīgaṇita and Triśatikā of Śrīdhara, Mahāvīra's Gaṇitasārasaṃgraha, Mahāsiddhānta of Āryabhata II, Siddhāntaśekhara and Gaṇitatilaka of Śrīpati, Līlāvatī of Bhāskara II, Gaṇitakaumudī of Nārāyaṇa and the Pañcaviṃśatikā, giving the correspondence in respect of numerous results. This would no doubt serve as a valuable reference to students and scholars alike.

The text of GSK consists of five parts, in the form of chapters. The authors note that the first three chapters are well structured like the compositions of his predecessors. The influence of Śrīdhara's Pāṭīgaṇita and Triśatikā, and Mahāvīra's Gaṇitasārasaṃgraha is discernible. The later chapters on the other hand involve what Pherū learnt from his own experience and perhaps oral literature of the time and Indo-Persian sources. The authors observe that Pherū aimed to produce a practical manual "useful for all professionals like bankers, traders, accountants and masons." The authors also point out that the supplementary material is valuable especially in that it offers a glimpse into the life of the Delhi-Haryana-Rajasthan region in the early fourteenth century as no other mathematical work does.

The first chapter of GSK, entitled "Twentyfive Fundamental Operations", begins with a discussion on weights and measures and the names of decimal places. Pherū's decimal reckoning goes all the way upto 10^{24} and is seen to be somewhat more systematised than other name lists; (it may be borne in mind that while naming of large number of powers of 10 has been a common feature in ancient Indian mathematics, the range covered, and also the names themselves have differed substantially, between traditions and over period). The remaining discussion in the chapter is about the arithmetical operations, including summation of strings of consecutive natural numbers, finding squares, square-roots, cubes and cube-roots of integers, a treatment of fractions, proportions (trairāśika and higher variations upto navarāśika and ekādaśarāśika; the ascendance is interesting, though towards the end the examples become rather too contorted!). The second chapter "Eight Classes of Fractions" deals, as the name suggests, with reduction of fractions and applications to practical contexts of distribution of a given sum into parts stipulated in various ways. The third chapter "Eight Types of Procedures" deals with arithmetic of mixtures, sums of strings of consecutive squares and cubes, topics in the geometry of the rectilinear figures and the circle, computations with shadows, volumes of various shapes, with many examples in the form of practical applications. For the area of a triangle Pherū gives Heron's formula and with it also Brahmagupta's generalisation of it to quadrilaterals; like in the Brāhmasphuṭasiddhānta here also it is

mentioned unconditionally (though the formula holds exactly only for cyclic quadrilaterals). The fact that it is not true for all quadrilaterals seems to have been missed by Pherū, though awareness of it is seen in the Mahāsiddhānta of Āryabhaṭa II and Līlavatī of Bhāskara II (see [2], Chapters 5 and 6). (Rather surprisingly the authors have not noted this point in the book, though it seems to be of some relevance from a historical point of view). Pherū describes rules for calculating the volumes of domes, towers in various shapes, arches, etc. which is substantially more elaborate compared to the earlier works; this may be correlated with the rise of Islamic architecture in India.

The fourth chapter "Four special topics", deals with certain special topics (confirming to the title). One of these is the topic of magic squares, and the authors note that Pherū's is the first discussion on the topic in mathematical text from India. It may be recalled here that comprehensive contributions to the topic were made later in the 14th century by Nārāyaṇa Paṇdita (see [2], Chapter 6). Pherū gives various constructions and classification of magic squares. Explicit examples of magic squares of orders upto 8 and 9 are also given. Another feature rather unique to the work is a discussion in this chapter of procedures for conversion of dates in the Vikrama years to the corresponding Hijrī dates and vice versa. There is also a section entitled "Cloth" dealing with the area of cloth needed in the construction of various types of tents, which were important for residential purpose at that time.

The relatively short last chapter in GSK is surmised not to belong to the original manuscript of GSK, but added by Pherū himself in later years. It deals with a variety of interesting practical topics such as yield of grains, yield of sugarcane juice and oil, yield from tax and masonry.

There has been growing interest in ancient Indian mathematics in recent years. The focus has however generally been on mathematics of the medieval period and the Kerala school originated by Madhava that flourished from around 1400 for about 200 years. The present book makes an important contribution towards well-rounding the subject, as it throws light on the mathematics during the emerging multicultural ethos of the early 14th century. It is well-written, well-produced and a joy to read and refer to.

REFERENCES

[1] SaKHYa, Gaṇitasārakaumudī; the Moonlight of the Essence of Mathematics, by Ṭhakkura Pherū; edited with Introduction, Translation, and Mathematical Commentary, Manohar Publishers, New Delhi 2009, xlvi+279 pp.

[2] Kim Plofker, Mathematics in India; Princeton University Press, Princeton, NJ, 2009, xiv+357 pp.

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