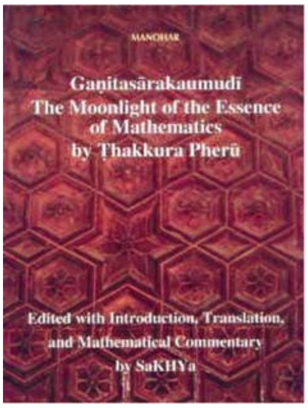

***Gaṇitasārakaumudī* – The Moonlight of the Essence of Mathematics.** Ṭhakkura Pherū, Edited with Introduction, Translation and Mathematical Commentary by SaKHYa (S. R. Sarma, T. Kusuba, T. Hayashi and M. Yano). Manohar Publishers and Distributors, 4753/23, Ansari Road, Daryaganj, New Delhi 110 002. 2009. viii + 278 pp. Price: Rs 995.

1. Consider several bonds, $i = 1, 2, \ldots, n$, which fetch interests f_i on a capital C for a standard duration, t. What is the average time and average interest, when we have several bonds corresponding to capitals C_i, earning interest F_i for time durations T_i, (when they are converted to a single bond)?

2. There are n gold pieces, whose weight and purity are w_i and v_i ($i = 1, 2, \ldots n$) respectively, smelted into a single alloy whose weight and purity are w and v. What is the relation among w_i, v_i, w and v?

3. 'A mother-in-law offers a confectionary called *varisola* to her n sons-in-law, and each one eats the same number of pieces. Every time before offering, she increases the number of pieces remaining on the plate by a_i ($i = 1, 2, \ldots, n$). The last man eats all the pieces on the plate. What is the original number of pieces (x_0) and what is the number of pieces (y) eaten by each person?'

4. What are the volume and surface area of the wall of a dome when its inner and outer circumference are given?

These are some of the problems which can be said to be in the genre of *vyavahāraganita* (that is, mathematics applied to day-to-day activities), which are discussed in *Gaṇitasārakaumudī* of Ṭhakkura Pherū, a work composed in the popular *apabhramsa* in early fourteenth century. Ṭhakkura Pherū belonged to the community of Srīmāla Jains noted for their expertise in minting and banking. He was employed in the treasury of Khilji sultans of Delhi. He is known for his works, *Ratnaparīksā* on gemology, *Jyotisasāra* on astronomy and astrology, *Vāstusāra* on architecture and iconography, and other works.

One of the authors of the present work (Sarma) published *Ratnaparīksā* with an English translation and commentary earlier. All the authors are well known indologists and have worked on various aspects of Indian mathematics and astronomy.

'*Gaṇitasārakaumudī* (GSK) is not only the first full-fledged mathematical text composed in *apabhramsa*, but it also extends the range of mathematics beyond the traditional framework of the earlier Sanskrit texts, and includes diverse topics from the daily life where numbers play a role.' In fact, Pheru was following the illustrious tradition of including topics on *vyavahāraganita* in mathematical works by the earlier writers like Mahavira and Sridhara. Indeed problems 1 and 2 above are discussed in Mahavira's *Ganitasārasamgraha* and Sridhara's *Pātīganita* and *Trisatikā*, all composed around 9th century AD. In fact, a large number of examples of arithmetical rules in GSK are adaptations of similar examples in Sridhara's *Pātīganita* and *Trisatikā*, and to a lesser extent of *Ganitasārasamgraha*.

GSK has five parts. The first three parts treat traditional topics of ordinary *pati* (arithmetic) works divided in two categories, namely, 'fundamental operations' (*parikramāni*) and 'procedures' (*vyavahārāh*). They include various fundamental operations including square, square root, cube, cube root, procedures for mixtures, proportional distributions, procedures for series, areas of figures, volumes of objects and capacities of pits of various geometries. The procedures for series include the sums of the first n integers, their squares and cubes. The value of π is taken to be 3, $\sqrt{10}$ or 3 + 1/6 at different places. The formulae for the surface area and volume of a sphere given here are not accurate, though Bhaskara-II had given the correct expressions for them in the 12th century. Many of the procedures and examples pertain to commercial transactions. 'Innovations in architecture which were being introduced by the Muslim rulers in this period find an echo in the section on solid geometry where Pherū lays down rules for calculating the volumes of domes, square and circular towers with spiral stairways, minarets with fluted columns, arches, bridges erected on supporting arches, and so on.'

Although the first three chapters of GSK are like the mathematical texts composed by his predecessors, the supplementary material in chapters 4 and 5 are different. Chapter 4 includes the classification and construction of magic squares. Earlier it was thought that it was Narayana Pandita who discussed magic squares for the first time in a mathematical work in his *Ganitakaumudī* (late fourteenth century). But GSK discusses magic squares mathematically including the 'knight rule' before *Ganitakaumudī*, and this is important from a historical view-point. 'The fifth chapter contains an interesting section which enumerates the average yield per bigha of several kinds of grains and pulses, the proportion of different products derived from sugarcane juice, and the amount of ghee that can be obtained from milk. This valuable data has naturally attracted the attention of economic historians.' According to the authors, 'the value of the *Gaṇitasārakaumudī* lies, to a large extent, in this supplementary material, which offers us a glimpse into the life of the Delhi–Haryana–Rajasthan region in the early fourteenth century as no other mathematical work does'.

Overall, GSK is not of the same class as *Pātīganita* of Sridhara or *Ganitasārasamgraha* of Mahavira, or *Ganitakaumudī* of Narayana, the last mentioned of which was composed a few decades after GSK. It appears that, whereas Sridhara *et al.* were primarily mathematicians who applied their mathematics to *vyavahāra*, Ṭhakkura Pherū was primarily a merchant/official who was conversant with mathematics of his times.

I noticed some errors in the explanations. The following problem is posed in 3.12: 'For one *damma*, one can get one *sera* of yellow myrobalan (*haradai*) or three seras of vibhītikā (*baheda*) or six seras of myrobalan (*āmalaya*). O physician (*vijja*), give for one *damma* equal amounts of all the three after grinding (*phakkiya*) them'. The answer given in p. 137 for the (equal) amounts is 3/2 *sera*s. It should have been 2/3 *sera*s. The prices would be 2/3, 2/9 and 1/9 *damma* for the *haradai*, *baheda* and *āmalaya*, and not 3/2, 1/2 and 1/4, as given.

The following problem (3.19) is discussed on p. 138: 'Calculate the purity of the gold alloy made from four gold pieces by refinement when $v_i = 9, 7, 10, 8$ *vanni*s, $w_i = 6, 5, 8, 7$ *tola*s respectively, and $W = 20$ *tolas*.' The answer given is $V = 9$ *vanni*s. It should have been 11¼ *vanni*s, obtained from a straightforward application of the formula $V = (\Sigma v_i w_i)/W$.

In problem (3.20) discussed on p. 138, one of the input values of w_i differs from the value specified by Pherū on p. 20 (and translated in p. 63). So, the answer given is also wrong. An input-error occurs also for the problem (1.91) (original verse, translation and explanation on p. 15, p. 55 and p. 123/124 respectively).

Ṭhakkura Pherū himself has committed a mistake in proposing a solution for the problem (3.24), to find the unknown weights of all the n component pieces (w_i) in a simple mixture (where the total weight of the mixture W is the sum of the weights of the component pieces w_i), when the purity of the individual components (v_i) and that of the mixture (V) are given. Pherū's answer is

$$w_i = a_i W \Big/ \left(\sum_{i=1}^{n} a_i \right),$$

where $a_i = 1/|V - v_i|$.

Considering the fact that there are only two relations among the weights w_i,

$$\sum_{i=1}^{n} w_i v_i = WV \quad \text{and} \quad \sum_{i=1}^{n} w_i = W,$$

the weights w_i are actually indeterminate except when $n = 2$ (two components). In that case, the given (correct) solution is the same as the one given by Mahavira in *Ganitasārasamgraha*. For $n > 2$, the equations are indeterminate. It can be shown that the rule is plainly wrong when n is odd. When n is even, the expressions for w_i correspond to one particular solution, when among the v_i, $n/2$ of them are greater than V, and $n/2$ of them are less than V. This is not commented upon in the explanatory notes, at all.

The present volume is a result of painstaking team-work of four well known researchers. It is praiseworthy that they took up a work composed in the popular *apabramsa*. All the scholarly norms of indologists are adhered to. Apart from an apt introduction, edited text and mathematical commentary, it has several appendices, indices, glossary and a comprehensive bibliography. It would be a very useful addition to any library which has a place for history of mathematics, or an individual collection of a serious scholar in the field. It has been brought out well by Manohar publishers also. However, I wish that the authors had addressed the book to non-scholars also (apart from indologists), who would be more interested in the contents of the verses and would like to know more about the Indian way of doing mathematics, and appreciate it. The explanations could have been more detailed, and at several places, diagrams would have been very helpful.

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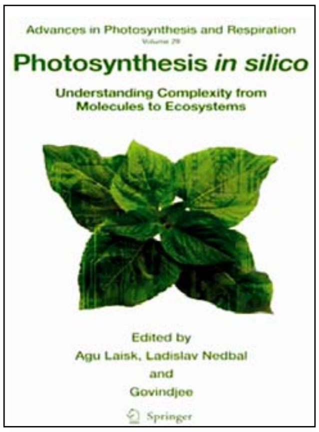

Advances in Photosynthesis and Respiration. Photosynthesis *in silico*: Understanding Complexity from Molecules to Ecosystems. Agu Laisk, Ladislav, Nedbal and Govindjee (eds). Springer, Dordrecht, The Netherlands. 2009. Vol. 29, 508 pp. Price: US$ 279.

The book deals with biological modelling of photosynthetic apparatus. Photosynthesis researchers are quite familiar with modelling. Many basic concepts of photosynthesis did emerge out of models including the Hill and Bendall Z-Scheme, the quencher Q-model of Duysens and Sweers, Kok's model for oxygen evolution, and even the Calvin–Benson cycle that won a Nobel Prize. However, these models provided mostly interpretation of results.

The current book is a new and massive effort to document global scale models and their complexities in photosynthesis. It aims at getting views from laboratory exercise to ecology. This volume is unique in many ways, not only in the content but also in concepts. This book explains events of photosynthesis in canopies, crop field, and grasslands and in nature. The preface of the book sets the tone and texture of the book. In 20 chapters, 44 experts belonging to 15 countries tell the readers what would be the shape of photosynthesis research in the future.

The volume contains five parts; the first part contains two chapters on the problem of modelling. Modelling usually involves a simplified visual representation of a system for understanding complex phenomena occurring within it. Data from experimental observation lead to information and information can be understood in terms of correlation between the entities existing within a system or subsystem. A model should not only explain the existing data but also predict future experimental results. Most of the biological models in earlier era were lacking association of mathematical equations. The models were qualitative representations. Moreover, there were no means to put together models of two different subsystems so that a model could emerge for the whole system. In the modern era the models include quantitative logical components. However, it remains a great challenge for understanding dynamic processes that are the essence of living systems. Computational methods appear to be the only medium to meet the challenge. The target can be achieved only if there are methods to merge e-models and grow them to a holistic model of biosystems. Achievement of this target demands platform-independent format for representing e-models. System Biology Markup Language (SBML) has been developed which can be helpful in achieving the target. Minimum Information Requested In the Annotation of biochemical Models (MIRIAM), System Biology Ontology (SBO) and Virtual Cell (VC) are the results of efforts in the area of biological modelling.

The second chapter discusses the application of computational tools in e-photosynthesis. Photosynthesis has its own complexity in terms of timescale stretch-